Hodgkin-Huxley Equations

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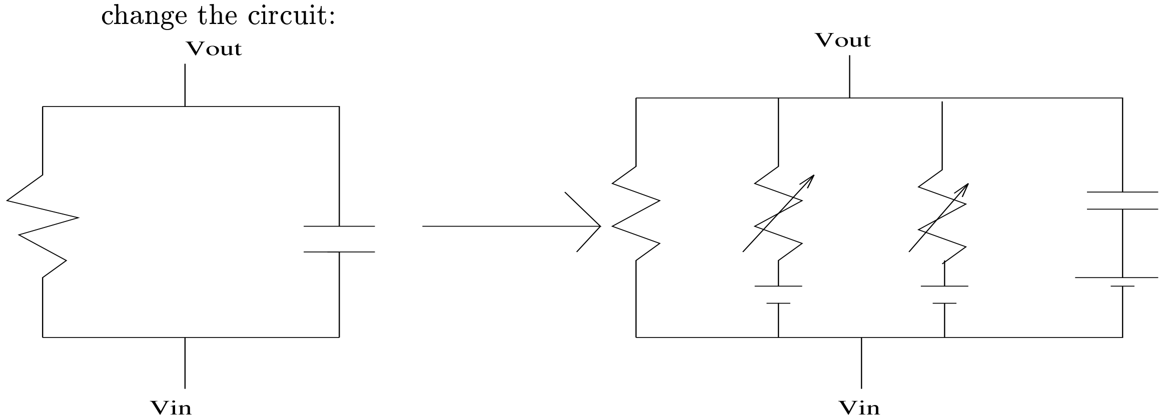
Abstract

We will see the Hodgkin-Huxley equations, the experiments, assumptions and analysis.

0.1 Definitions

- Remember Cable Theory.
- The ecuation for the PASIVE cable are:

As we know the axon is not passive. It has active conductances. Let's change the circuit:



Now, we have a cable with variable conductances. The problem is to find the new cable equation. This can be done, but we will only get to the variation of voltatage with respect to time.

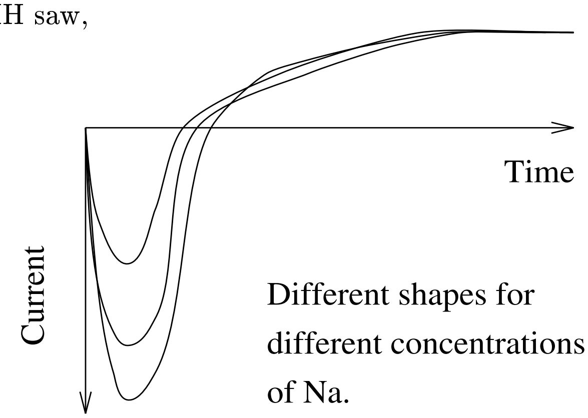
The total current in the capacitance of the cable is

$$I = C_m \frac{dV}{dt} + I_i$$

where I_i are the currents due to the variable conductances. If we apply a Voltage Clamp to the system $\frac{dV}{dt} = 0$, consequently,

$$I = I_i$$

this is what HH saw,



In order to study the dependence of this current with respect to sodium and potassium, HH bathe the axon in different concentration of Na. They saw, that when there were no Na, the inward part of the current did not exists. They performed all their experiments with 30experiments (sea water).

Using this, and the Nernst equation, they could corroborate the Na resting potential and be sure that the inward part of the current was due to it.

Let's define the prime letters to be the variables of the 30and the unprime to be the control.

Assumptions

- Assumption 1. The function $I_k(t)$ is independent of $[Na]_o$. This is $I_k(t) = I'_k(t)$.
- Assumption 2. If $I_{Na}(t)$ and $I'_{Na}(t)$ then

$$\frac{I'_{Na}(t)}{I_{Na}(t)} = k$$

- Assumption 3. Let $[t_0, t_0 + T]$ denote the interval during which $I_{Na}(t)$ increases to its masimum value. We assume that if $t \in [t_0, t_0 + T/3]$ or $t_0 \leq t \leq t_0 + T/3$, then

$$\frac{dI_K}{dt}(t) = 0$$

Now we also are under the assumption that

$$I_i = I_{Na} + I_K$$

and

$$I_i' = I_{Na}' + I_K'$$

easely we can get to

$$I_{Na}(t) = \frac{I_i(t) - I_i'(t)}{1-k}$$

and

$$I_k'(t) = \frac{I_i' - kI_i}{1-k}$$

0.2 The good stuff

Analyzing again the equation of ion currents:

$$I_i = I_{Na} + I_K + I_l$$

where I_l is the leakeage current. Using the concept of conductance and that the currents are separable we have that

$$I_{Na} = gNa(E - E_{Na})$$

$$I_K = gK(E - E_K)$$

where gNa and gK are variables. And it turns to be that

$$I_l = \bar{g}_l(E - E_{Na})$$

with $\bar{g}_l$ a constatn (we will use this as the notation).

The whole current equation is

$$I = C_m \frac{dV}{dt} + g_{Na}V_{Na} + g_k V_k + \bar{g}_l V_l$$

or, which is the same

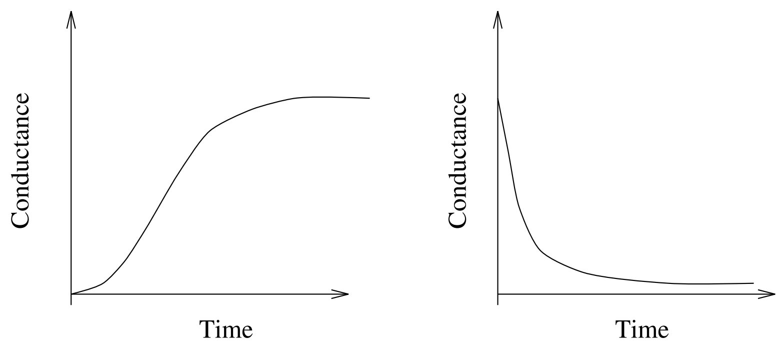
$$\frac{dV}{dt} = \frac{1}{C_m}[I - g_{Na}V_{Na} - g_k V_k - \bar{g}_l V_l]$$

this is the most famous form of the HH equation. Now, we have to determine the behavior of the conductances.

The procedure is standard. Let's analyze the temporal behavior of each conductance and then try to fit a function.

- For potassium.

Conductance of Potassium in Voltage Clamp



and this is the touchstone of the theory, they propose a dimensionless variable n.

$$g_K = \bar{g}_K[n(t)]^4$$

that satisfy the differential equation

$$\frac{dn}{dt} = \alpha_n(1-n)\beta_n n$$

with α and β funcitons of V. Experimentaly, we can determine the value of this variables using the next new variables,

$$\tau_n = \frac{1}{\alpha_n + \beta_n}$$

$$n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n}$$

where n_∞ is the value of n after applying a nonzero voltage clamp, and can be determined by

$$\bar{g}_K n_\infty^4 = g_\infty$$

determination of τ_n is more complicated, but also can be obtaned from experiments. With this two number we can get α_n and β_n The final result is

$$\alpha_n(V) = \frac{0.01(V+10)}{exp[(V+10)/10]-1}$$

$$\beta_n(V) = 0.125exp(V/80)$$

- For sodium. It is the same idea, but more complicated.

$$g_{Na} = m^3 h \bar{g}_{Na}$$

$$\frac{dm}{dt} = \alpha_m(1-m) - \beta_m m$$

$$\frac{dh}{dt} = \alpha_h(1-h) - \beta_h h$$

$$\alpha_m = \frac{0.1(V+25)}{exp[(V+25)/10-1}$$

$$\beta_m = 4exp[\frac{V}{18}]$$

$$\alpha_h = 0.07exp(\frac{V}{20})$$

$$\beta_h = \frac{1}{exp[(V+30)/10]+1}$$

0.3 The analysis

- Good predictor of current clamp.
- Godd predictor of anode break.
- When applying constant current (enough) the system will oscillate, and that is not what it happens.
- When applying a ramp of current the system will oscillate, and again, that is not true.