Chaos and Dynamical Systems

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December 2, 1995

Abstract

In this paper I present the definitions of chaos and dynamical systems. Also the ways to identify a chaotic system and examples.

0.1 Definitions

- Dynamical system: Set of differential equations related by one variable (parameter). Usually time.

- Chaotic system:

    - A determinic system that seems to be a probabilistic system.
    - A dynamical system in which it's Lyapunov coeffitients are negative.
    - Irregular oscillatory process.
    - Mathematicaly we only need 2 differential equations to get chaos.

There are three funtamental characteristics of chaos. a) irregular periodicity, b) sensityivity to initial conditions, and c) a lack of predictability.
Examples of chaotic systems.

- Forced pendulum

- Lasers

- Chemical reactions (Zhabotinky's Oscillator)

- Biological models for population dynamics

- Stimulated heart cells

Famous chaotic sysemts.

- Lorentz equations

- Hopf bifurcation

- Logistic map

Phase plane.

- Phase plane is f' vs. f

- Attractor. Final state of the system when initial conditions are near the that point.

- Stange attractor. You will know when you see it.

Energy and dimensions.

- Lyapunov function. Is the energy function of our system.

- Lyapunov's coeffitients are the eigenvalues of the energy.

- Correlation dimnsion. Degrees of freedom of our system. If the dimension is an integer, then the system id not chaotic, otherwise it is. (think this as the earth is moving in a 2D plane but a little in 3D).

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